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COMMENT

A comment on the reducibility of the Voigt functions

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Abstract. An elementary derivation of the expression for the Voigt functions in terms of circular functions and confluent hypergeometric functions is presented, correcting some errors in Exton's recent analysis.

An expression for the Voigt functions

$$K(x, y)=\pi^{-1/2}\int_0^\infty \exp(-yr-\tfrac{1}{4}r^2)\cos(xr)\,\mathrm{d}r \tag{1}$$

and

$$L(x, y)=\pi^{-1/2}\int_0^\infty \exp(-yr-\tfrac{1}{4}r^2)\sin(xr)\,\mathrm{d}r \tag{2}$$

in terms of circular functions and confluent hypergeometric functions was recently derived by Exton (1981a). His derivation involves the use of generalised hypergeometric functions.

An elementary derivation can be obtained by noting that

$$\begin{aligned}K(x, y)-\mathrm{i}L(x, y)&\\ &=\pi^{-1/2}\int_0^\infty \exp[-(y+\mathrm{i}x)r-\tfrac{1}{4}r^2]\,\mathrm{d}r\\ &=\exp[(y+\mathrm{i}x)^2]\,\mathrm{erfc}(y+\mathrm{i}x) \qquad \text{(AS 7.4.2)}\\ &=\exp[(y+\mathrm{i}x)^2]-2(y+\mathrm{i}x)\pi^{-1/2}{}_1F_1(1;\tfrac{3}{2};(y+\mathrm{i}x)^2) \qquad \text{(AS 7.1.2).}\end{aligned} \tag{3}$$

Here (AS a.b.c) stands for identity a.b.c in Abramowitz and Stegun (1965).

$K(x, y)$, the real part of the above expression, is therefore expressible as

$$\begin{aligned}K(x, y)=\exp(y^2-x^2)\cos(2xy)-\pi^{-1/2}[(y+\mathrm{i}x)\,{}_1F_1(1;\tfrac{3}{2};(y+\mathrm{i}x)^2)&\\ +(y-\mathrm{i}x)\,{}_1F_1(1;\tfrac{3}{2};(y-\mathrm{i}x)^2)].&\end{aligned} \tag{4}$$

Similarly, for $L(x, y)$ we obtain

$$\begin{aligned}L(x, y)=-\exp(y^2-x^2)\sin(2xy)+\pi^{-1/2}[(x+\mathrm{i}y)\,{}_1F_1(1;\tfrac{3}{2};-(x+\mathrm{i}y)^2)&\\ +(x-\mathrm{i}y)\,{}_1F_1(1;\tfrac{3}{2};-(x-\mathrm{i}y)^2)].&\end{aligned} \tag{5}$$

To write these expressions in a form more similar to Exton's equations (21) and (22), we note that $(y+\mathrm{i}x)^2=-(x-\mathrm{i}y)^2$, and $(y-\mathrm{i}x)^2=-(x+\mathrm{i}y)^2$, and that

$$2z\ {}_1F_1(1;\tfrac{3}{2};z)={}_1F_1(1;\tfrac{1}{2};z)-1 \qquad \text{(AS 13.4.6).}$$

It is clear that our equations (5) and (6) differ from Exton's principal results.

The well known special cases (Haubold and John 1979)

$$K(0,y)=\exp(y^2)-\pi^{-1/2}\,2y\ {}_1F_1(1;\tfrac{3}{2};y^2)$$

$$L(0,y)=0$$

$$K(x,0)=\exp(-x^2)$$

and

$$L(x,0)=\pi^{-1/2}\,2x\ {}_1F_1(1;\tfrac{3}{2};-x^2)$$

all agree with our equations (5) and (6), but except for $L(0,y)$, they do not agree with Exton's equations (21) and (22), indicating that the latter are not quite correct.

Exton (1981b) has pointed out that the errors in equations (21) and (22) of Exton (1981a) are due to an error in equation (9), there, whose right-hand member should read

$$\psi_2(1;\tfrac{3}{2},\tfrac{1}{2};-x^2,y^2)-\ \ldots .$$

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